Chapter 18 Review Questions and Problems, page 785

11. $\Phi = 10(1 - x + y), \quad F = 10 - 10(1 + i)z$

13. $\Phi = \text{Re}\,(220 - 95.54\,\text{Ln}\,z) = 220 - \dfrac{220}{\ln 10}\ln r = 220 - 95.54 \ln r.$

17. $2(1 - (2/\pi)\,\text{Arg}\,z)$

19. $30(1 - (2/\pi)\,\text{Arg}\,(z - 1))$

21. $\Phi = x + y = \text{const}, \quad V = \overline{F'(z)} = 1 - i$, parallel flow

23. $T(x, y) = x(2y + 1) = \text{const}$

25. $\overline{F'(z)} = \bar{z} + 1 = x + 1 - iy$

Problem Set 19.1, page 796

1. $0.84175 \cdot 10^2, \quad -0.52868 \cdot 10^3, \quad 0.92414 \cdot 10^{-3}, \quad -0.36201 \cdot 10^6$

3. 6.3698, 6.794, 8.15, impossible

5. Add first, then round.

7. 29.9667, 0.0335; 29.9667, 0.0333704 (6S-exact)

9. 29.97, 0.035; 29.97, 0.03337; 30, 0.0; 30, 0.033

11. $|\epsilon| = |x + y - (\tilde{x} + \tilde{y})| = |(x - \tilde{x}) + (y - \tilde{y})| = |\epsilon_x + \epsilon_y|$
$\leqq |\epsilon_x| + |\epsilon_y| = \beta_x + \beta_y$

13. $\dfrac{a_1}{a_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2 + \epsilon_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2}\left(1 - \dfrac{\epsilon_2}{\tilde{a}_2} + \dfrac{\epsilon_2^2}{\tilde{a}_2^2} - + \cdots\right) \approx \dfrac{\tilde{a}_1}{\tilde{a}_2} + \dfrac{\epsilon_1}{\tilde{a}_2} - \dfrac{\epsilon_2}{\tilde{a}_2} \cdot \dfrac{\tilde{a}_1}{\tilde{a}_2},$

hence $\left|\left(\dfrac{a_1}{a_2} - \dfrac{\tilde{a}_1}{\tilde{a}_2}\right)\Big/\dfrac{a_1}{a_2}\right| \approx \left|\dfrac{\epsilon_1}{a_1} - \dfrac{\epsilon_2}{a_2}\right| \leqq |\epsilon_{r1}| + |\epsilon_{r2}| \leqq \beta_{r1} + \beta_{r2}$

15. **(a)** $1.38629 - 1.38604 = 0.00025$, **(b)** $\ln 1.00025 = 0.000249969$ is 6S-exact.

19. In the present case, (b) is slightly more accurate than (a) (which may produce nonsensical results; cf. Prob. 20).

21. $c_4 \cdot 2^4 + \cdots + c_0 \cdot 2^0 = (1\,0\,1\,1\,1.)_2$, NOT $(1\,1\,1\,0\,1.)_2$

23. The algorithm in Prob. 22 repeats 0011 infinitely often.

25. $n = 26$. The beginning is 0.09375 ($n = 1$).

27. $I_{14} = 0.1812$ (0.1705 4S-exact), $\quad I_{13} = 0.1812$ (0.1820), $\quad I_{12} = 0.1951$ (0.1951), $I_{11} = 0.2102$ (0.2103), etc.

29. $-0.126 \cdot 10^{-2}, -0.402 \cdot 10^{-3}; \quad -0.266 \cdot 10^{-6}, -0.847 \cdot 10^{-7}$

Problem Set 19.2, page 807

3. $g = 0.5 \cos x, \quad x = 0.450184$ ($= x_{10}$, exact to 6S)

5. Convergence to 4.7 for all these starting values.

7. $x = x/(e^x \sin x)$; 0.5, 0.63256, $\cdots$ converges to 0.58853 (5S-exact) in 14 steps.

9. $x = x^4 - 0.12; \quad x_0 = 0, x_3 = -0.119794$ (6S-exact)

11. $g = 4/x + x^3/16 - x^5/576; \quad x_0 = 2, x_n = 2.39165$ ($n \geqq 6$), 2.405 4S-exact

13. This follows from the intermediate value theorem of calculus.

15. $x_3 = 0.450184$

17. Convergence to $x = 4.7, 4.7, 0.8, -0.5$, respectively. Reason seen easily from the graph of f.

19. 0.5, 0.375, 0.377968, 0.377964; (b) $1/\sqrt{7}$

21. 1.834243 $(= x_4)$, 0.656620 $(= x_4)$, -2.49086 $(= x_4)$

23. $x_0 = 4.5$, $x_4 = 4.73004$ (6S-exact)

25. (a) ALGORITHM BISECT (f, a_0, b_0, ϵ, N) Bisection Method

This algorithm computes the solution c of $f(x) = 0$ (f continuous) within the tolerance ϵ, given an initial interval $[a_0, b_0]$ such that $f(a_0)f(b_0) < 0$.

INPUT: Continuous function f, initial interval $[a_0, b_0]$, tolerance ϵ, maximum number of iterations N.

OUTPUT: A solution c (within the tolerance ϵ), or a message of failure.

For $n = 0, 1, \cdots, N - 1$ do:

$c = \frac{1}{2}(a_n + b_n)$

If $f(c) = 0$ then OUTPUT c Stop. [*Procedure completed*]

Else if $f(a_n)f(b_n) < 0$ then set $a_{n+1} = a_n$ and $b_{n+1} = c$.

Else set $a_{n+1} = c$, and $b_{n+1} = b_n$.

If $|a_{n+1} - b_{n+1}| < \epsilon|c|$ then OUTPUT c. Stop. [*Procedure completed*]

End

OUTPUT $[a_N, b_N]$ and a message "Failure". Stop.

[*Unsuccessful completion; N iterations did not give an interval of length not exceeding the tolerance.*]

End BISECT

Note that $[a_N, b_N]$ gives $(a_N + b_N)/2$ as an approximation of the zero and $(b_N - a_N)/2$ as a corresponding error bound.

(b) 0.739085; **(c)** 1.30980, 0.429494

27. $x_2 = 1.5$, $x_3 = 1.76471, \cdots$, $x_7 = 1.83424$ (6S-exact)

29. 0.904557 (6S-exact)

Problem Set 19.3, page 819

1. $L_0(x) = -2x + 19$, $L_1(x) = 2x - 18$, $p_1(9.3) = L_0(9.3) \cdot f_0 + L_1(9.3) \cdot f_1$ $= 0.1086 \cdot 9.3 + 1.230 = 2.2297$

3. $$p_2(x) = \frac{(x - 1.02)(x - 1.04)}{(-0.02)(-0.04)} \cdot 1.0000 + \frac{(x - 1)(x - 1.04)}{0.02(-0.02)} \cdot 0.9888 + \frac{(x - 1)(x - 1.02)}{0.04 \cdot 0.02} \cdot 0.9784 = x^2 - 2.580x + 2.580;$$ 0.9943, 0.9835

5. 0.8033 (error -0.0245), 0.4872 (error -0.0148); quadratic: 0.7839 (-0.0051), 0.4678 (0.0046)

7. $p_2(x) = 1.1640x - 0.3357x^2$; -0.5089 (error 0.1262), 0.4053 (-0.0226), 0.9053 (0.0186), 0.9911 (-0.0672)

9. $p_2(x) = -0.44304x^2 + 1.30896x - 0.023220$, $p_2(0.75) = 0.70929$ (5S-exact 0.71116)

11. $L_0 = -\frac{1}{6}(x - 1)(x - 2)(x - 3)$, $L_1 = \frac{1}{2}x(x - 2)(x - 3)$, $L_2 = -\frac{1}{2}x(x - 1)(x - 3)$, $L_3 = \frac{1}{6}x(x - 1)(x - 2)$; $p_3(x) = 1 + 0.039740x - 0.335187x^2 + 0.060645x^3$; $p_2(0.5) = 0.943654$, $p_3(1.5) = 0.510116$, $p_3(2.5) = -0.047991$

13. $2x^2 - 4x + 2$

15. $p_3(x) = 2.1972 + (x - 9) \cdot 0.1082 + (x - 9)(x - 9.5) \cdot 0.005235$

17. $r = -1.5$, $p_2(0.3) = 0.6039 + (-1.5) \cdot 0.1755 + \frac{1}{2}(-1.5)(-0.5) \cdot (-0.0302)$ $= 0.3293$

Problem Set 19.4, page 826

9. $[-1.39(x-5)^2 + 0.58(x-5)^3]'' = 0.004$ at $x = 5.8$ (due to roundoff; should be 0).

11. $1 - \frac{5}{4}x^2 + \frac{1}{4}x^4$

13. $1 - x^2$, $-2(x-1) - (x-1)^2 + 2(x-1)^3$, $-1 + 2(x-2) + 5(x-2)^2 - 6(x-2)^3$

15. $4 + x^2 - x^3$, $-8(x-2) - 5(x-2)^2 + 5(x-2)^3$, $4 + 32(x-4) + 25(x-4)^2 - 11(x-4)^3$

17. Use the fact that the third derivative of a cubic polynomial is constant, so that g''' is piecewise constant, hence constant throughout under the present assumption. Now integrate three times.

19. Curvature $f''/(1 + f'^2)^{3/2} \approx f''$ if $|f'|$ is small.

Problem Set 19.5, page 839

1. 0.747131, which is larger than 0.746824. Why?

3. 0.5, 0.375, 0.34375, 0.335 (exact)

5. $\epsilon_{0.5} \approx 0.03452$ ($\epsilon_{0.5} = 0.03307$), $\epsilon_{0.25} \approx 0.00829$ ($\epsilon_{0.25} = 0.00820$)

7. 0.693254 (6S-exact 0.693147)

9. 0.073930 (6S-exact 0.073928)

11. 0.785392 (6S-exact 0.785398)

13. $(0.785398126 - 0.785392156)/15 = 0.39792 \cdot 10^{-6}$

15. **(a)** $M_2 = 2$, $|KM_2| = 2/(12n^2) = 10^{-5}/2$, $n = 183$. **(b)** $f^{\text{iv}} = 24/x^5$, $M_4 = 24$, $|CM_4| = 24/(180 \cdot (2m)^4) = 10^{-5}/2$, $2m = 12.8$, hence 14.

17. 0.94614588, 0.94608693 (8S-exact 0.94608307)

19. 0.9460831 (7S-exact)

21. 0.9774586 (7S-exact 0.9774377)

23. Set $x = \frac{1}{2}(t+1)$, 0.2642411177 (10S-exact), $1 - 2/e$

25. $x = \frac{1}{2}(t+1)$, $dx = \frac{1}{2}dt$, 0.746824127 (9S-exact 0.746824133)

27. 0.08, 0.32, 0.176, 0.256 (exact)

29. $5(0.1040 - \frac{1}{2} \cdot 0.1760 + \frac{1}{3} \cdot 0.1344 - \frac{1}{4} \cdot 0.0384) = 0.256$

Chapter 19 Review Questions and Problems, page 841

17. 4.375, 4.50, 6.0, impossible

19. $44.885 \leqq s \leqq 44.995$

21. The same as that of $\tilde{a}$.

23. $x = 20 \pm \sqrt{398} = 20.00 \pm 19.95$, $x_1 = 39.95$, $x_2 = 0.05$, $x_2 = 2/39.95 = 0.05006$ (error less than 1 unit of the last digit)

25. $x = x^4 - 0.1$, -0.1, -0.999, -0.99900399

27. 0.824

29. $-x + x^3$, $2(x-1) + 3(x-1)^2 - (x-1)^3$

31. 0.26, $M_2 = 6$, $M_2^* = 0$, $-0.02 \leqq \epsilon \leqq 0$, 0.01

33. 0.90443, 0.90452 (5S-exact 0.90452)

35. (a) $(0.4^3 - 2 \cdot 0.2^3 + 0)/0.04 = 1.2$, (b) $(0.3^3 - 2 \cdot 0.2^3 + 0.1^3)/0.01 = 1.2$ (exact)

Problem Set 20.1, page 851

1. $x_1 = 7.3, \quad x_2 = -3.2$ **3.** No solution **5.** $x_1 = 2, \quad x_2 = 1$

7. $\begin{bmatrix} -3 & 6 & -9 & -46.725 \\ 0 & 9 & -13 & -51.223 \\ 0 & 0 & -2.88889 & -7.38689 \end{bmatrix}$

$x_1 = 3.908, \quad x_2 = -1.998, \quad x_3 = 2.557$

9. $\begin{bmatrix} 13 & -8 & 0 & 178.54 \\ 0 & 6 & 13 & 137.86 \\ 0 & 0 & -16 & -253.12 \end{bmatrix}$

$x_1 = 6.78, \quad x_2 = -11.3, \quad x_3 = 15.82$

11. $\begin{bmatrix} 3.4 & -6.12 & -2.72 & 0 \\ 0 & 0 & 4.32 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = t_1$ arbitrary, $\quad x_2 = (3.4/6.12)t_1, \quad x_3 = 0$

13. $\begin{bmatrix} 5 & 0 & 6 & -0.329193 \\ 0 & -4 & -3.6 & -2.143144 \\ 0 & 0 & 2.3 & -0.4 \end{bmatrix}$

$x_1 = 0.142856, \quad x_2 = 0.692307, \quad x_3 = -0.173912$

15. $\begin{bmatrix} -1 & -3.1 & 2.5 & 0 & -8.7 \\ 0 & 2.2 & 1.5 & -3.3 & -9.3 \\ 0 & 0 & -1.493182 & -0.825 & 1.03773 \\ 0 & 0 & 0 & 6.13826 & 12.2765 \end{bmatrix}$

$x_1 = 4.2, \quad x_2 = 0, \quad x_3 = -1.8, \quad x_4 = 2.0$

Problem Set 20.2, page 857

1. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}, \quad \begin{matrix} x_1 = -4 \\ x_2 = 6 \end{matrix}$

3. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad \begin{matrix} x_1 = 0.4 \\ x_2 = 0.8 \\ x_3 = 1.6 \end{matrix}$

5. $\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 3 & 9 & 1 \end{bmatrix} \begin{bmatrix} 3 & 9 & 6 \\ 0 & -6 & 3 \\ 0 & 0 & -3 \end{bmatrix}, \quad \begin{matrix} x_1 = -\frac{1}{15} \\ x_2 = \frac{4}{15} \\ x_3 = \frac{2}{5} \end{matrix}$

7. $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, $x_1 = 0.6$, $x_2 = 1.2$, $x_3 = 0.4$

9. $\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}\begin{bmatrix} 0.1 & 0 & 0.3 \\ 0 & 0.4 & 0.2 \\ 0 & 0 & 0.1 \end{bmatrix}$, $x_1 = 2$, $x_2 = -11$, $x_3 = 4$

11. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 3 & -1 & 3 & 0 \\ 2 & 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, $x_1 = 2$, $x_2 = -3$, $x_3 = 4$, $x_4 = -1$

13. No, since $\mathbf{x}^\mathsf{T}(-\mathbf{A})\mathbf{x} = -\mathbf{x}^\mathsf{T}\mathbf{A}\mathbf{x} < 0$; yes; yes; no

15. $\begin{bmatrix} -3.5 & 1.25 \\ 3.0 & -1.0 \end{bmatrix}$

17. $\frac{1}{36}\begin{bmatrix} 584 & 104 & -66 \\ 104 & 20 & -12 \\ -66 & -12 & 9 \end{bmatrix}$

19. $\frac{1}{16}\begin{bmatrix} 21 & -6 & -14 & 6 \\ -6 & 36 & -12 & -4 \\ -14 & -12 & 20 & -4 \\ 6 & -4 & -4 & 4 \end{bmatrix}$

Problem Set 20.3, page 863

5. Exact 0.5, 0.5, 0.5

7. $x_1 = 2$, $x_2 = -4$, $x_3 = 8$

9. Exact 2, 1, 4

11. **(a)** $\mathbf{x}^{(3)\mathsf{T}} = [0.49983 \quad 0.50001 \quad 0.500017]$,
(b) $\mathbf{x}^{(3)\mathsf{T}} = [0.50333 \quad 0.49985 \quad 0.49968]$

13. 8, −16, 43, 86 steps; spectral radius 0.09, 0.35, 0.72, 0.85, approximately

15. $[1.99934 \quad 1.00043 \quad 3.99684]^\mathsf{T}$ (Jacobi, Step 5); $[2.00004 \quad 0.998059 \quad 4.00072]^\mathsf{T}$ (Gauss–Seidel)

19. $\sqrt{306} = 17.49$, 12, 12

Problem Set 20.4, page 871

1. 18, $\sqrt{110} = 10.49$, 8, $[0.125 \quad -0.375 \quad 1 \quad 0 \quad -0.75 \quad 0]$

3. 5.9, $\sqrt{13.81} = 3.716$, 3, $\frac{1}{3}[0.2 \quad 0.6 \quad -2.1 \quad 3.0]$

5. 5, $\sqrt{5}$, 1, $[1 \quad 1 \quad 1 \quad 1 \quad 1]$

7. $ab + bc + ca = 0$

9. $\kappa = 5 \cdot \frac{1}{2} = 2.5$ **11.** $\kappa = (5 + \sqrt{5})(1 + 1/\sqrt{5}) = 6 + 2\sqrt{5}$

13. $\kappa = 19 \cdot 13 = 247$; ill-conditioned

15. $\kappa = 20 \cdot 20 = 400$; ill-conditioned

17. $167 \leqq 21 \cdot 15 = 315$

19. $[-2 \quad 4]^\mathsf{T}$, $[-144.0 \quad 184.0]^\mathsf{T}$, $\kappa = 25{,}921$, extremely ill-conditioned

21. Small residual $[0.145 \quad 0.120]$, but large deviation of $\tilde{\mathbf{x}}$.

23. 27, 748, 28,375, 943,656, 29,070,279

Problem Set 20.5, page 875

1. $1.846 - 1.038x$ **3.** $1.48 + 0.09x$

5. $s = 90t - 675$, $v_{\text{av}} = 90$ km/hr **9.** $-11.36 + 5.45x - 0.589x^2$

11. $1.89 - 0.739x + 0.207x^2$

13. $2.552 + 16.23x$, $-4.114 + 13.73x + 2.500x^2$, $2.730 + 1.466x - 1.778x^2 + 2.852x^3$

Problem Set 20.7, page 884

1. 5, 0, 7; radii 6, 4, 6. Spectrum $\{-1, 4, 9\}$

3. Centers 0; radii 0.5, 0.7, 0.4. Skew-symmetric, hence $\lambda = i\mu$, $-0.7 \leqq \mu \leqq 0.7$.

5. 2, 3, 8; radii $1 + \sqrt{2}$, 1, $\sqrt{2}$; actually (4S) 1.163, 3.511, 8.326

7. $t_{11} = 100$, $t_{22} = t_{33} = 1$

9. They lie in the intervals with endpoints $a_{jj} \pm (n-1) \cdot 10^{-5}$. Why?

11. $\rho(\mathbf{A}) \leqq$ Row sum norm $\|\mathbf{A}\|_\infty = \max_j \sum_k |a_{jk}| = \max_j(|a_{jj}| +$ Gerschgorin radius$)$

13. $\sqrt{122} = 11.05$

15. $\sqrt{0.52} = 0.7211$

17. Show that $\mathbf{A}\bar{\mathbf{A}}^\mathsf{T} = \bar{\mathbf{A}}^\mathsf{T}\mathbf{A}$.

19. 0 lies in no Gerschgorin disk, by (3) with $>$; hence $\det \mathbf{A} = \lambda_1 \cdots \lambda_n \neq 0$.

Problem Set 20.8, page 887

1. $q = 10, 10.9908, 10.9999$; $|\epsilon| \leqq 3, 0.3028, 0.0275$

3. $q \pm \delta = 4 \pm 1.633$, 4.786 ± 0.619, 4.917 ± 0.398

5. Same answer as in Prob. 3, possibly except for small roundoff errors.

7. $q = 5.5, 5.5738, 5.6018$; $|\epsilon| \leqq 0.5, 0.3115, 0.1899$; eigenvalues (4S) 1.697, 3.382, 5.303, 5.618

9. $\mathbf{y} = \mathbf{A}\mathbf{x} = \lambda\mathbf{x}$, $\mathbf{y}^\mathsf{T}\mathbf{x} = \lambda\mathbf{x}^\mathsf{T}\mathbf{x}$, $\mathbf{y}^\mathsf{T}\mathbf{y} = \lambda^2\mathbf{x}^\mathsf{T}\mathbf{x}$, $\epsilon^2 \leqq \mathbf{y}^\mathsf{T}\mathbf{y}/\mathbf{x}^\mathsf{T}\mathbf{x} - (\mathbf{y}^\mathsf{T}\mathbf{x}/\mathbf{x}^\mathsf{T}\mathbf{x})^2 = \lambda^2 - \lambda^2 = 0$

11. $q = 1, \cdots, -2.8993$ approximates -3 (0 of the given matrix), $|\epsilon| \leqq 1.633, \cdots, 0.7024$ (Step 8)

Problem Set 20.9, page 896

1. $$\begin{bmatrix} 0.98 & -0.4418 & 0 \\ -0.4418 & 0.8702 & 0.3718 \\ 0 & 0.3718 & 0.4898 \end{bmatrix}$$

3. $\begin{bmatrix} 7 & -3.6056 & 0 \\ -3.6056 & 13.462 & 3.6923 \\ 0 & 3.6923 & 3.5385 \end{bmatrix}$

5. $\begin{bmatrix} 3 & -67.59 & 0 & 0 \\ -67.59 & 143.5 & 45.35 & 0 \\ 0 & 45.35 & 23.34 & 3.126 \\ 0 & 0 & 3.126 & -33.87 \end{bmatrix}$

7. Eigenvalues 16, 6, 2

$\begin{bmatrix} 11.2903 & -5.0173 & 0 \\ -5.0173 & 10.6144 & 0.7499 \\ 0 & 0.7499 & 2.0952 \end{bmatrix}, \begin{bmatrix} 14.9028 & -3.1265 & 0 \\ -3.1265 & 7.0883 & 0.1966 \\ 0 & 0.1966 & 2.0089 \end{bmatrix}, \begin{bmatrix} 15.8299 & -1.2932 & 0 \\ -1.2932 & 6.1692 & 0.0625 \\ 0 & 0.0625 & 2.0010 \end{bmatrix}$

9. Eigenvalues (4S) 141.4, 68.64, −30.04

$\begin{bmatrix} 141.1 & 4.926 & 0 \\ 4.926 & 68.97 & 0.8691 \\ 0 & 0.8691 & -30.03 \end{bmatrix}, \begin{bmatrix} 141.3 & 2.400 & 0 \\ 2.400 & 68.72 & 0.3797 \\ 0 & 0.3797 & -30.04 \end{bmatrix}, \begin{bmatrix} 141.4 & 1.166 & 0 \\ 1.166 & 68.66 & 0.1661 \\ 0 & 0.1661 & -30.04 \end{bmatrix}$

Chapter 20 Review Questions and Problems, page 896

15. $[3.9 \quad 4.3 \quad 1.8]^T$
17. $[-2 \quad 0 \quad 5]^T$

19. $\begin{bmatrix} 0.28193 & -0.15904 & -0.00482 \\ -0.15904 & 0.12048 & -0.00241 \\ -0.00482 & -0.00241 & 0.01205 \end{bmatrix}$

21. $\begin{bmatrix} 5.750 \\ 3.600 \\ 0.838 \end{bmatrix}, \begin{bmatrix} 6.400 \\ 3.559 \\ 1.000 \end{bmatrix}, \begin{bmatrix} 6.390 \\ 3.600 \\ 0.997 \end{bmatrix}$
Exact: $[6.4 \quad 3.6 \quad 1.0]^T$

23. $\begin{bmatrix} 1.700 \\ 1.180 \\ 4.043 \end{bmatrix}, \begin{bmatrix} 1.986 \\ 0.999 \\ 4.002 \end{bmatrix}, \begin{bmatrix} 2.000 \\ 1.000 \\ 4.000 \end{bmatrix}$
Exact: $[2 \quad 1 \quad 4]^T$

25. 42, $\sqrt{674} = 25.96$, 21
27. 30
29. 5
31. $115 \cdot 0.4458 = 51.27$
33. $5 \cdot \frac{21}{63} = \frac{5}{3}$
35. $1.514 + 1.129x - 0.214x^2$
37. Centers 15, 35, 90; radii 30, 35, 25, respectively. Eigenvalues (3S) 2.63, 40.8, 96.6
39. Centers 0, −1, −4; radii 9, 6, 7, respectively; eigenvalues 0, 4.446, −9.446

Problem Set 21.1, page 910

1. $y = 5e^{-0.2x}$, 0.00458, 0.00830 (errors of y_5, y_{10})
3. $y = x - \tanh x$ (set $y - x = u$), 0.00929, 0.01885 (errors of y_5, y_{10})
5. $y = e^x$, 0.0013, 0.0042 (errors of y_5, y_{10})
7. $y = 1/(1 - x^2/2)$, 0.00029, 0.01187 (errors of y_5, y_{10})
9. Errors 0.03547 and 0.28715 of y_5 and y_{10} much larger
11. $y = 1/(1 - x^2/2)$; error -10^{-8}, $-4 \cdot 10^{-8}$, $\cdots$, $-6 \cdot 10^{-7}$, $+9 \cdot 10^{-6}$; $\epsilon = 0.0002/15 = 1.3 \cdot 10^{-5}$ (use RK with $h = 0.2$)
13. $y = \tan x$; error $0.83 \cdot 10^{-7}, 0.16 \cdot 10^{-6}, \cdots, -0.56 \cdot 10^{-6}, +0.13 \cdot 10^{-5}$
15. $y = 3 \cos x - 2 \cos^2 x$; error $\cdot 10^7$: 0.18, 0.74, 1.73, 3.28, 5.59, 9.04, 14.3, 22.8, 36.8, 61.4
17. $y' = 1/(2 - x^4)$; error $\cdot 10^9$: 0.2, 3.1, 10.7, 23.2, 28.5, -32.3, -376, -1656, -3489, $+80444$
19. Errors for Euler–Cauchy 0.02002, 0.06286, 0.05074; for improved Euler–Cauchy -0.000455, 0.012086, 0.009601; for Runge–Kutta. 0.0000011, 0.000016, 0.000536

Problem Set 21.2, page 915

1. $y = e^x$, $y_5^* = 1.648717$, $y_5 = 1.648722$, $\epsilon_5 = -3.8 \cdot 10^{-8}$, $y_{10}^* = 2.718276$, $y_{10} = 2.718284$, $\epsilon_{10} = -1.8 \cdot 10^{-6}$
3. $y = \tan x$, $y_4, \cdots, y_{10}$ (error $\cdot 10^5$) 0.422798 (-0.49), 0.546315 (-1.2), 0.684161 (-2.4), 0.842332 (-4.4), 1.029714 (-7.5), 1.260288 (-13), 1.557626 (-22)
5. RK error smaller in absolute value, error $\cdot 10^5 = 0.4, 0.3, 0.2, 5.6$ (for $x = 0.4, 0.6, 0.8, 1.0$)
7. $y = 1/(4 + e^{-3x})$, $y_4, \cdots, y_{10}$ (error $\cdot 10^5$) 0.232490 (0.34), 0.236787 (0.44), 0.240075 (0.42), 0.242570 (0.35), 0.244453 (0.25), 0.245867 (0.16), 0.246926 (0.09)
9. $y = \exp(x^3) - 1$, $y_4, \cdots, y_{10}$ (error $\cdot 10^7$) 0.008032 (-4), 0.015749 (-10), 0.027370 (-17), 0.043810 (-26), 0.066096 (-39), 0.095411 (-54), 0.133156 (-74)
13. $y = \exp(x^2)$. Errors $\cdot 10^5$ from $x = 0.3$ to 0.7: $-5, -11, -19, -31, -41$
15. **(a)** 0, 0.02, 0.0884, 0.215848, $y_4 = 0.417818$, $y_5 = 0.708887$ (poor)
(b) By 30–50%

Problem Set 21.3, page 922

1. $y_1 = -e^{-2x} + 4e^x$, $y_2 = -e^{-2x} + e^x$; errors of y_1 (of y_2) from 0.002 to 0.5 (from -0.01 to 0.1), monotone
3. $y_1' = y_2$, $y_2' = -\frac{1}{4}y_1$, $y = y_1 = 1$, 0.99, 0.97, 0.94, 0.9005, error $-0.005, -0.01, -0.015, -0.02, -0.0229$; exact $y = \cos \frac{1}{2}x$
5. $y_1' = y_2$, $y_2' = y_1 + x$, $y_1(0) = 1$, $y_2(0) = -2$, $y = y_1 = e^{-x} - x$, $y = 0.8$ (error 0.005), 0.61 (0.01), 0.429 (0.012), 0.2561 (0.0142), 0.0905 (0.0160)
7. By about a factor 10^5. $\epsilon_n(y_1) \cdot 10^6 = -0.082, \cdots, -0.27$, $\epsilon_n(y_2) \cdot 10^6 = 0.08, \cdots, 0.27$
9. Errors of y_1 (of y_2) from $0.3 \cdot 10^{-5}$ to $1.3 \cdot 10^{-5}$ (from $0.3 \cdot 10^{-5}$ to $0.6 \cdot 10^{-5}$)
11. $(y_1, y_2) = (0, 1), (0.20, 0.98), (0.39, 0.92), \cdots, (-0.23, -0.97), (-0.42, -0.91), (-0.59), (-0.81)$; continuation will give an "ellipse."

Problem Set 21.4, page 930

3. $-3u_{11} + u_{12} = -200, \quad u_{11} - 3u_{12} = -100$
5. 105, 155, 105, 115; Step 5: 104.94, 154.97, 104.97, 114.98
7. 0, 0, 0, 0. All equipotential lines meet at the corners (why?). Step 5: 0.29298, 0.14649, 0.14649, 0.073245
9. 0.108253, 0.108253, 0.324760, 0.324760; Step 10: 0.108538, 0.108396, 0.324902, 0.324831
11. **(a)** $u_{11} = -u_{12} = -66$. **(b)** Reduce to 4 equations by symmetry.
$u_{11} = u_{31} = -u_{15} = -u_{35} = -92.92$, $u_{21} = -u_{25} = -87.45$,
$u_{12} = u_{32} = -u_{14} = -u_{34} = -64.22$, $u_{22} = -u_{24} = -53.98$,
$u_{13} = u_{23} = u_{33} = 0$
13. $u_{12} = u_{32} = 31.25, \quad u_{21} = u_{23} = 18.75, \quad u_{jk} = 25$ at the others
15. $u_{21} = u_{23} = 0.25, \quad u_{12} = u_{32} = -0.25, \quad u_{jk} = 0$ otherwise
17. $\sqrt{3}$, $u_{11} = u_{21} = 0.0849$, $u_{12} = u_{22} = 0.3170$. (0.1083, 0.3248 are 4S-values of the solution of the linear system of the problem.)

Problem Set 21.5, page 935

5. $u_{11} = 0.766, \quad u_{21} = 1.109, \quad u_{12} = 1.957$, $u_{22} = 3.293$
7. **A**, as in Example 1, right sides −220, −220, −220, −220. Solution $u_{11} = u_{21} = 125.7$, $u_{21} = u_{22} = 157.1$
13. $-4u_{11} + u_{21} + u_{12} = -3$, $u_{11} - 4u_{21} + u_{22} = -12$, $u_{11} - 4u_{12} + u_{22} = 0$, $2u_{21} + 2u_{12} - 12u_{22} = -14$, $u_{11} = u_{22} = 2$, $u_{21} = 4$, $u_{12} = 1$.
Here $-\frac{14}{3} = -\frac{4}{3}(1 + 2.5)$ with $\frac{4}{3}$ from the stencil.
15. $\mathbf{b} = [-200, -100, -100, 0]^\mathsf{T}$; $\quad u_{11} = 73.68$, $u_{21} = u_{12} = 47.37$, $u_{22} = 15.79$ (4S)

Problem Set 21.6, page 941

5. 0, 0.6625, 1.25, 1.7125, 2, 2.1, 2, 1.7125, 1.25, 0.6625, 0
7. Substantially less accurate, 0.15, 0.25 ($t = 0.04$), 0.100, 0.163 ($t = 0.08$)
9. Step 5 gives 0, 0.06279, 0.09336, 0.08364, 0.04707, 0.
11. Step 2: 0 (exact 0), 0.0453 (0.0422), 0.0672 (0.0658), 0.0671 (0.0628), 0.0394 (0.0373), 0 (0)
13. 0.3301, 0.5706, 0.4522, 0.2380 ($t = 0.04$), 0.06538, 0.10603, 0.10565, 0.6543 ($t = 0.20$)
15. 0.1018, 0.1673, 0.1673, 0.1018 ($t = 0.04$), 0.0219, 0.0355, $\cdots$ ($t = 0.20$)

Problem Set 21.7, page 944

1. $u(x, 1) = 0, -0.05, -0.10, -0.15, -0.20, 0$
3. For $x = 0.2, 0.4$ we obtain 0.24, 0.40 ($t = 0.2$), 0.08, 0.16 ($t = 0.4$), −0.08, −0.16 ($t = 0.6$), etc.
5. 0, 0.354, 0.766, 1.271, 1.679, 1.834, $\cdots$ ($t = 0.1$); 0, 0.575, 0.935, 1.135, 1.296, 1.357, $\cdots$ ($t = 0.2$)
7. 0.190, 0.308, 0.308, 0.190, (3S-exact: 0.178, 0.288, 0.288, 0.178)

Chapter 21 Review Questions and Problems, page 945

17. $y = e^x$, 0.038, 0.125 (errors of y_5 and y_{10})
19. $y = \tan x$; 0 (0), 0.10050 (−0.00017), 0.20304 (−0.00033), 0.30981 (−0.00048), 0.42341 (−0.00062), 0.54702 (−0.00072), 0.68490 (−0.00076), 0.84295 (−0.00066), 1.0299 (−0.0002), 1.2593 (0.0009), 1.5538 (0.0036)
21. 0.1003346 $(0.8 \cdot 10^{-7})$ 0.2027099 $(1.6 \cdot 10^{-7})$, 0.3093360 $(2.1 \cdot 10^{-7})$, 0.4227930 $(2.3 \cdot 10^{-7})$, 0.5463023 $(1.8 \cdot 10^{-7})$
23. $y = \sin x$, $y_{0.8} = 0.717366$, $y_{1.0} = 0.841496$ (errors $-1.0 \cdot 10^{-5}$, $-2.5 \cdot 10^{-5}$)
25. $y_1' = y_2$, $y_2' = x^2 y_1$, $y = y_1 = 1, 1, 1, 1.0001, 1.0006, 1.002$
27. $y_1' = y_2$, $y_2' = 2e^x - y_1$, $y = e^x - \cos x$, $y = y_1 = 0, 0.241, 0.571, \cdots$; errors between 10^{-6} and 10^{-5}
29. 3.93, 15.71, 58.93
31. 0, 0.04, 0.08, 0.12, 0.15, 0.16, 0.15, 0.12, 0.08, 0.04, 0 ($t = 0.3$. 3 time steps)
33. $u(P_{11}) = u(P_{31}) = 270$, $u(P_{21}) = u(P_{13}) = u(P_{23}) = u(P_{33}) = 30$, $u(P_{12}) = u(P_{32}) = 90$, $u(P_{22}) = 60$
35. 0.043330, 0.077321, 0.089952, 0.058488 ($t = 0.04$), 0.010956, 0.017720, 0.017747, 0.010964 ($t = 0.20$)

Problem Set 22.1, page 953

3. $f(\mathbf{x}) = 2(x_1 - 1)^2 + (x_2 + 2)^2 - 6$; Step 3: (1.037, −1.926), value −5.992
9. Step 5: (0.11247, −0.00012), value 0.000016

Problem Set 22.2, page 957

7. No
9. x_3, x_4 is the unused time on M_1, M_2, respectively.
11. $f(2.5, 2.5) = 100$
13. $f(-\frac{11}{3}, \frac{26}{3}) = 198\frac{1}{3}$
15. $f(9, 6) = 360$
17. $0.5x_1 + 0.75x_2 \leqq 45$ (copper), $0.5x_1 + 0.25x_2 \leqq 30$, $f = 120x_1 + 100x_2$, $f_{\max} = f(45, 30) = 8400$
19. $f = x_1 + x_2$, $2x_1 + 3x_2 \leqq 1200$, $4x_1 + 2x_2 \leqq 1600$, $f_{\max} = f(300, 200) = 500$
21. $x_1/3 + x_2/2 \leqq 100$, $x_1/3 + x_2/6 \leqq 80$, $f = 150x_1 + 100x_2$, $f_{\max} = f(210, 60) = 37{,}500$

Problem Set 22.3, page 961

3. $f(120/11, 60/11) = 480/11$
5. Eliminate in Column 3, so that 20 goes. $f_{\min} = f(0, \frac{1}{2}) = -10$.
7. $f_{\max} = f(\frac{60}{21}, 0, \frac{1500}{105}, 0) = \frac{2200}{7}$
9. $f_{\max} = 6$ on the segment from (3, 0, 0) to (0, 0, 2)
11. We minimize! The augmented matrix is

$$\mathbf{T}_0 = \begin{bmatrix} 1 & 1.8 & 2.1 & 0 & 0 & 0 \\ 0 & 15 & 30 & 1 & 0 & 150 \\ 0 & 600 & 500 & 0 & 1 & 3900 \end{bmatrix}.$$

The pivot is 600. The calculation gives

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & \frac{6}{10} & 0 & -\frac{3}{1000} & -\frac{117}{10} \\ 0 & 0 & \frac{35}{2} & 1 & -\frac{1}{40} & \frac{105}{2} \\ 0 & 600 & 500 & 0 & 1 & 3900 \end{bmatrix} \quad \begin{matrix} \text{Row 1} - \frac{1.8}{600}\text{ Row 3} \\ \text{Row 2} - \frac{15}{600}\text{ Row 3} \\ \text{Row 3} \end{matrix}$$

The next pivot is $\frac{35}{2}$. The calculation gives

$$\mathbf{T}_2 = \begin{bmatrix} 1 & 0 & 0 & -\frac{6}{175} & -\frac{3}{1400} & -\frac{27}{2} \\ 0 & 0 & \frac{35}{2} & 1 & -\frac{1}{40} & \frac{105}{2} \\ 0 & 600 & 0 & -\frac{200}{7} & \frac{12}{7} & 2400 \end{bmatrix} \quad \begin{matrix} \text{Row 1} - \frac{1.2}{35}\text{ Row 2} \\ \text{Row 2} \\ \text{Row 3} - \frac{1000}{35}\text{ Row 2} \end{matrix}$$

Hence $-f$ has the maximum value -13.5, so that f has the minimum value 13.5, at the point

$$(x_1, x_2) = \left(\frac{2400}{600}, \frac{105/2}{35/2}\right) = (4, 3).$$

13. $f_{\max} = f(5, 4, 6) = 478$

Problem Set 22.4, page 968

1. $f(6, 3) = 84$
3. $f(20, 20) = 40$
5. $f(10, 5) = 5500$
7. $f(1, 1, 0) = 13$
9. $f(4, 0, \frac{1}{2}) = 9$

Chapter 22 Review Questions and Problems, page 968

9. Step 5: $[0.353 \quad -0.028]^{\mathsf{T}}$. Slower. Why?
11. Of course! Step 5: $[-1.003 \quad 1.897]^{\mathsf{T}}$
17. $f(2, 4) = 100$
19. $f(3, 6) = -54$

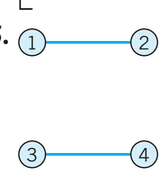
Problem Set 23.1, page 974

9. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

15.



17. If G is complete.

19. Vertex / Edge

Vertex	e_1	e_2	e_3	e_4
1	-1	-1	1	-1
2	1	0	0	0
3	0	1	-1	0
4	0	0	0	1

Problem Set 23.2, page 979

1. 5

3. 4

5. The idea is to go backward. There is a v_{k-1} adjacent to v_k and labeled $k - 1$, etc. Now the only vertex labeled 0 is s. Hence $\lambda(v_0) = 0$ implies $v_0 = s$, so that $v_0 - v_1 - \cdots - v_{k-1} - v_k$ is a path $s \to v_k$ that has length k.

15. Delete the edge (2, 4).

17. No

Problem Set 23.3, page 983

1. (1, 2), (2, 4), (4, 3); $L_2 = 12, L_3 = 36, L_4 = 28$

5. (1, 2), (2, 4), (3, 4), (3, 5); $L_2 = 2, L_3 = 4, L_4 = 3, L_5 = 6$

7. (1, 2), (2, 4), (3, 4); $L_2 = 10, L_3 = 15, L_4 = 13$

9. (1, 5), (2, 3), (2, 6), (3, 4), (3, 5); $L_2 = 9, L_3 = 7, L_4 = 8, L_5 = 4, L_6 = 14$

Problem Set 23.4, page 987

1.
```
2\
   4 − 3 − 5     L = 10
1/
```

3.
```
             1
            /
5 − 3 − 6 <            L = 17
            \
             2 − 4
```

5.
```
     2
    /
1 <      3     L = 12
    \   /
     4 <
        \
         5
```

9. Yes

11.
```
             2
            /
1 − 3 − 4 <            L = 38
            \
             5 − 6
```

13. New York–Washington–Chicago–Dalles–Denver–Los Angeles

15. G is connected. If G were not a tree, it would have a cycle, but this cycle would provide two paths between any pair of its vertices, contradicting the uniqueness.

19. If we add an edge (u, v) to T, then since T is connected, there is a path $u \to v$ in T which, together with (u, v), forms a cycle.

Problem Set 23.5, page 990

1. If G is a tree.
3. A shortest spanning tree of the largest connected graph that contains vertex 1.
7. (1, 4), (1, 3), (1, 2), (2, 6), (3, 5); $L = 32$
9. (1, 4), (4, 3), (4, 2), (3, 5); $L = 20$
11. (1, 4), (4, 3), (4, 5), (1, 2); $L = 12$

Problem Set 23.6, page 997

1. $\{3, 6\}$, $11 + 3 = 14$
3. $\{4, 5, 6\}$, $10 + 5 + 13 = 28$
5. $\{3, 6, 7\}$, $8 + 4 + 4 = 16$
7. $S = \{1, 4\}$, $8 + 6 = 14$
9. One is interested in flows *from s to t,* not in the opposite direction.
13. $\Delta_{12} = 5, \Delta_{24} = 8, \Delta_{45} = 2$; $\Delta_{12} = 5, \Delta_{25} = 3$; $\Delta_{13} = 4, \Delta_{35} = 9$
P_1: $1 - 2 - 4 - 5, \Delta f = 2$; P_2: $1 - 2 - 5, \Delta f = 3$; P_3: $1 - 3 - 5, \Delta f = 4$
15. $1 - 2 - 5, \Delta f = 2$; $1 - 4 - 2 - 5, \Delta f = 2$, etc.
17. $f_{13} = f_{35} = 8$, $f_{14} = f_{45} = 5$, $f_{12} = f_{24} = f_{46} = 4$, $f_{56} = 13$, $f = 4 + 13 = 17$, $f = 17$ is unique.
19. For instance, $f_{12} = 10$, $f_{24} = f_{45} = 7$, $f_{13} = f_{25} = 5$, $f_{35} = 3$, $f_{32} = 2$, $f = 3 + 5 + 7 = 15$, $f = 15$ is unique.

Problem Set 23.7, page 1000

3. (2, 3) and (5, 6)
5. By considering only edges with one labeled end and one unlabeled end
7. $1 - 2 - 5, \Delta_t = 2$; $1 - 4 - 2 - 5, \Delta_t = 1$; $f = 6 + 2 + 1 = 9$, where 6 is the given flow
9. $1 - 2 - 4 - 6, \Delta_t = 2$; $1 - 3 - 5 - 6, \Delta_t = 1$; $f = 4 + 2 + 1 = 7$, where 4 is the given flow
15. $S = \{1, 2, 4, 5\}$, $T = \{3, 6\}$, $\text{cap}(S, T) = 14$

Problem Set 23.8, page 1005

1. No
3. No
5. Yes, $S = \{1, 4, 5, 8\}$
7. Yes, $S = \{1, 3, 5\}$
11. $1 - 2 — 3 - 7 — 5 - 4$
13. $1 - 2 — 3 - 7 — 5 - 4$ is augmenting and gives $1 — 2 - 3 — 7 - 5 — 4$ and (1, 2), (3, 7), (5, 4) is of maximum cardinality.
15. $1 - 4 — 3 - 6 — 7 - 8$ is augmenting and gives $1 — 4 - 3 — 6 - 7 — 8$ and (1, 4), (3, 6), (7, 8) is of maximum cardinality.
19. 3
21. 2
23. 3
25. K_4

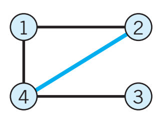
Chapter 23 Review Questions and Problems, page 1006

11. $$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

13.

From vertex \ To vertex	1	2	3	4
1	0	1	0	1
2	1	0	1	0
3	0	1	0	1
4	1	0	1	0

15.



17.

Vertex	Incident Edges
1	(1, 2), (1, 4)
2	(2, 1), (2, 4)
3	(3, 4)
4	(4, 1), (4, 2), (4, 3)

19. (1, 2), (1, 4), (2, 3); $L_2 = 2, L_3 = 5, L_4 = 5$

23. (1, 6), (4, 5), (2, 3), (7, 8)

Problem Set 24.1, page 1015

1. $q_L = 19, q_M = 20, q_U = 20.5$

3. $q_L = 138, q_M = 144, q_U = 154$

5. $q_L = 199, q_M = 201, q_U = 201$

7. $q_L = 1.3, q_M = 1.4, q_U = 1.45$

9. $q_L = 89.9, q_M = 91.0, q_U = 91.8$

11. $\bar{x} = 19.875, s = 0.835$, IQR $= 1.5$

13. $\bar{x} = 144.67, s = 8.9735$, IQR $= 16$

15. $\bar{x} = 1.355, s = 0.136$, IQR $= 0.15$

17. 3.54, 1.29

Problem Set 24.2, page 1017

1. 2^3 outcomes: *RRR, RRL, RLR, LRR, RLL, LRL, LLR, LLL*

3. $6^2 = 36$ outcomes $(1, 1), (1, 2), \cdots, (6, 6)$, first number (second number) referring to the first die (second die)

5. Infinitely many outcomes $H \quad TH \quad TTH \quad TTTH \quad \cdots \quad (H = Head, T = Tail)$

7. The space of ordered pairs of numbers

9. 10 outcomes: $D \quad ND \quad NND \quad \cdots \quad NNNNNNNNND$

11. Yes

17. $A \cup B = B$ implies $A \subseteq B$ by the definition of union. Conversely. $A \subseteq B$ implies that $A \cup B = B$ because always $B \subseteq A \cup B$, and if $A \subseteq B$, we must have equality in the previous relation.

Problem Set 24.3, page 1024

1. $1 - 4/216 = 98.15\%$, by Theorem 1
3. **(a)** $0.9^3 = 72.9\%$, **(b)** $\frac{90}{100} \cdot \frac{89}{99} \cdot \frac{88}{98} = 72.65\%$
5. $\frac{8}{9}$
7. Small sample from a large population containing *many* items in each class we are interested in (defectives and nondefectives, etc.)
9. $\frac{498}{500} \cdot \frac{497}{499} \cdot \frac{496}{498} \cdot \frac{495}{497} \cdot \frac{494}{496} \approx 0.98008$
11. **(a)** $\frac{100}{200} \cdot \frac{99}{199} = 24.874\%$, **(b)** $\frac{100}{200} \cdot \frac{100}{199} + \frac{100}{200} \cdot \frac{100}{199} = 50.25\%$, **(c)** same as (a). (a) + (b) + (c) = 1. Why?
13. $1 - 0.96^3 = 11.5\%$
15. $1 - 0.875^4 = 0.4138 < 1 - 0.75^2 = 0.4375 < 0.5 \quad (c < b < a)$
17. $A = B \cup (A \cap B^c)$, hence $P(A) = P(B) + P(A \cap B^c) \geqq P(B)$ by disjointedness of B and $A \cap B^c$

Problem Set 24.4, page 1028

1. In $10! = 3{,}628{,}800$ ways
3. $\frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} = \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{2}{2} \cdot \frac{1}{1} = \frac{4!2!}{6!} = \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{15}$
5. $\binom{10}{3}\binom{5}{2}\binom{6}{2} = 18{,}000$
7. 210, 70, 112, 28
9. In $6!/6 = 120$ ways
11. $9 \cdot 8 = 72$
13. **(b)** $1/(12n)$
15. P (*No two people have a birthday in common*) $= 365 \cdot 364 \cdots 346/365^{20} = 0.59$. *Answer:* 41%, which is surprisingly large.

Problem Set 24.5, page 1034

1. $k = \frac{1}{55}$ by (6)
3. $k = \frac{1}{4}$ by (10), $P(0 \leqq X \leqq 2) = \frac{1}{2}$
5. No, because of (6)
7. $k = \frac{1}{100}$ because of (6) and $1 + 8 + 27 + 64 = 100$
9. $k = 5$; 50%
11. $0.5^3 = 12.5\%$
13. $F(x) = 0$ if $x < -1$, $F(x) = \frac{1}{2}(x + 1)^2$ if $-1 \leqq x < 0$
$F(x) = 1 - \frac{1}{2}(x - 1)^2$ if $0 \leqq x < 1$, $F(x) = 1$ if $x \leqq 1$
Answer: 500 cans, $P = 0.125, 0$
15. $X > b, X \geqq b, X < c, X \leqq c$, etc.

Problem Set 24.6, page 1038

1. $k = \frac{1}{2}, \mu = \frac{4}{3}, \sigma^2 = \frac{2}{9}$
3. $\mu = \pi, \sigma^2 = \pi^2/3$; cf. Example 2
5. $\mu = \frac{1}{4}, \sigma^2 = \frac{1}{16}$
7. $C = \frac{1}{2}, \mu = 2, \sigma^2 = 4$
9. 750, 1, 0.002
11. $c = 0.073$
13. \$643.50
15. $\frac{1}{2}, \frac{1}{20}, (X - \frac{1}{2})\sqrt{20}$
17. X = *Product of the 2 numbers*. $E(X) = 12.25$, 12 cents
19. $(0 + 1 \cdot 3 + 3 \cdot 8 + 1 \cdot 27)/8 = 54/8 = 6 \cdot 75$

Problem Set 24.7, page 1044

3. 38%
5. $\binom{5}{x} 0.5^5$, 0.03125, 0.15625, $1 - f(0) = 0.96875$, 0.96875
7. 0.265
9. $f(x) = 0.5^x e^{-0.5}/x!$, $f(0) + f(1) = e^{-0.5}(1.0 + 0.5) = 0.91$. *Answer:* 9%
11. $13\frac{1}{4}$%
13. 42%, 47.2%, 10.5%, 0.3%
15. $1 - e^{-0.2} = 18\%$

Problem Set 24.8, page 1050

1. 0.1587, 0.5, 0.6915, 0.6247
3. 45.065, 56.978, 2.022
5. 15.9%
7. 31.1%, 95.4%
9. About 58%
11. $t = 1084$ hours
13. About 683 (Fig. 521a)

Problem Set 24.9, page 1059

1. $\frac{1}{8}, \frac{3}{16}, \frac{3}{8}$
3. $\frac{2}{9}, \frac{1}{9}, \frac{1}{2}$
5. $f_2(y) = 1/(\beta_2 - \alpha_2)$ if $\alpha_2 < y < \beta_2$
7. 27.45 mm, 0.38 mm
11. 25.26 cm, 0.0078 cm
13. 50%
15. The distributions in Prob. 17 and Example 1
17. No

Chapter 24 Review Questions and Problems, page 1060

11. $Q_L = 110$, $Q_M = 112$, $Q_U = 115$
13. $\bar{x} = 111.9$, $s = 4.0125$, $s^2 = 16.1$
21. $x_{\text{min}} \leqq x_j \leqq x_{\text{max}}$. Sum over j from 1.
17. $\bar{x} = 6$, $s = 3.65$
19. $f(x) = \binom{50}{x} 0.03^x 0.97^{50-x} \approx 1.5^x e^{-1.5}/x!$
21. $f(x) = 2^{-x}$, $x = 1, 2, \cdots$
23. $1, \frac{1}{2}$
25. 0.1587, 0.6306, 0.5, 0.4950

Problem Set 25.2, page 1067

1. In Example 1, $\mu = 0$ so $\sum_{j=1}^{n} x_j = 0$. $\partial \ln \ell/\partial \ell = 0$ and $\tilde{\sigma}^2$ is as before.
3. $\ell = e^{-n\mu}\mu^{(x_1 + \cdots + x_n)}/(x_1! \cdots x_n!)$, $\partial \ln \ell/\partial \mu = -n + (x_1 + \cdots + x_n)/\mu = 0$, $n\hat{\mu} = n\bar{x}$, $\hat{\mu} = \bar{x} = 15.3$
5. $l = p^k(1 - p)^{n-k}$, $\hat{p} = k/n$, $k =$ number of successes in n trails
7. 7/12
9. $l = f = p(1 - p)^{x-1}$, etc., $\hat{p} = 1/x$
11. $\hat{\theta} = n/\Sigma x_j = 1/\bar{x}$
13. $\hat{\theta} = 1$
15. Variability larger than perhaps expected

Problem Set 25.3, page 1077

3. Shorter by a factor $\sqrt{2}$

5. 4, 16

7. $c = 1.96, \bar{x} = 126, s^2 = 126 \cdot 674/800 = 106.155, k = cs/\sqrt{n} = 0.714,$
$\text{CONF}_{0.95}\{125.3 \leqq \mu \leqq 126.7\}, \text{CONF}_{0.95}\{0.1566 \leqq p \leqq 0.1583\}$

9. $\text{CONF}_{0.99}\{63.72 \leqq \mu \leqq 66.28\}$

11. $n - 1 = 5, F(c) = 0.995, c = 4.03, \bar{x} = 9533.33, s^2 = 49{,}666.67,$
$k = 366.66$ (Table 25.2), $\text{CONF}_{0.99}\{9166.7 \leqq \mu \leqq 9900\}$

13. $\text{CONF}_{0.95}\{0.023 \leqq \sigma^2 \leqq 0.085\}$

15. $n - 1 = 99$ degrees of freedom. $F(c_1) = 0.025,\ c_1 = 74.2,\ F(c_2) = 0.975,$
$c_2 = 129.6$. Hence $k_1 = 12.41, k_2 = 7.10$. $\text{CONF}_{0.95}\{7.10 \leqq \sigma^2 \leqq 12.41\}$.

17. $\text{CONF}_{0.95}\{0.74 \leqq \sigma^2 \leqq 5.19\}$

19. $Z = X + Y$ is normal with mean 105 and variance 1.25.
Answer: $P(104 \leqq Z \leqq 106) = 63\%$

Problem Set 25.4, page 1086

3. $t = (0.286 - 0)/(4.31/\sqrt{7}) = 0.18 < c = 1.94$; accept the hypothesis.

5. $c = 6090 > 6019$: do not reject the hypothesis.

7. $\sigma^2/n = 1.8, c = 57.8$, accept the hypothesis.

9. $\mu < 58.69$ or $\mu > 61.31$

11. Alternative $\mu \neq 5000, t = (4990 - 5000)/(20/\sqrt{50}) = -3.54 < c = -2.01$
(Table A9, Appendix 5). Reject the hypothesis $\mu = 5000$ g.

13. Two-sided. $t = (0.55 - 0)/\sqrt{0.546/8} = 2.11 < c = 2.37$ (Table A9, Appendix 5),
no difference

15. $19 \cdot 1.0^2/0.8^2 = 29.69 < c = 30.14$ (Table A10. Appendix 5), accept the hypothesis

17. By (12), $t_0 = \sqrt{16}(20.2 - 19.6)/\sqrt{0.16 + 0.36} > c = 1.70$. Assert that B is better.

Problem Set 25.5, page 1091

1. $\text{LCL} = 1 - 2.58 \cdot 0.02/2 = 0.974, \text{UCL} = 1.026$

3. 27

5. Choose 4 times the original sample size

9. $2.58\sqrt{0.0004}/\sqrt{2} = 0.036, \text{LCL} = 3.464, \text{UCL} = 3.536$

11. $\text{LCL} = np - 3\sqrt{np(1-p)}, \text{CL} = np, \text{UCL} = np + 3\sqrt{np(1-p)}$

13. In about 30% (5%) of the cases

15. $\text{LCL} = \mu - 3\sqrt{\mu}$ is negative in (b) and we set $\text{LCL} = 0, \text{CL} = \mu = 3.6,$
$\text{UCL} = \mu + 3\sqrt{\mu} = 9.3$.

Problem Set 25.6, page 1095

1. 0.9825, 0.9384, 0.4060

3. 0.8187, 0.6703, 0.1353

5. $e^{-25\theta}(1 + 25\theta), P(A; 1.5) = 94.5, \alpha = 5.5\%$

7. 19.5%, 14.7%

9. $(1 - \theta)^n + n\theta(1 - \theta)^{n-1}$

11. $(1 - \frac{1}{2})^3 + 3 \cdot \frac{1}{2}(1 - \frac{1}{2})^2 = \frac{1}{2}$

13. $\sum_{x=0}^{9} \binom{100}{x} 0.12^x\, 0.88^{100-x} = 22\%$ (by the normal approximation)

15. $(1 - \theta)^5, [\theta(1 - \theta)^{5-1}]' = 0, \theta = \frac{1}{6}, \text{AOQL} = 6.7\%$

Problem Set 25.7, page 1099

3. $\chi_0^2 = (40 - 50)^2/50 + (60 - 50)^2/50 = 4 > c = 3.84$; no
5. $\chi_0^2 = \frac{16}{10} > 11.07$; yes
7. $\chi_0^2 = 10.264 < 11.07$; yes
9. 42 even digits, accept.
13. $\chi_0^2 = \dfrac{(355 - 358.5)^2}{358.5} + \dfrac{(123 - 119.5)^2}{119.5} = 0.137 < c = 3.84$ (1 degree of freedom, 95%)
15. Combining the last three nonzero values, we have $K - r - 1 = 9$ ($r = 1$ since we estimated the mean, $\frac{10{,}094}{2608} \approx 3.87$). $\chi_0^2 = 12.8 < c = 16.92$. Accept the hypothesis.

Problem Set 25.8, page 1102

3. $(\frac{1}{2})^8 + 8 \cdot (\frac{1}{2})^8 = 3.5\%$ is the probability that 7 cases in 8 trials favor A under the hypothesis that A and B are equally good. Reject.
5. $(\frac{1}{2})^{18}(1 + 18 + 153 + 816) = 0.0038$
7. $\bar{x} = 9.67$, $s = 11.87$. $t_0 = 9.67/(11.87/\sqrt{15}) = 3.16 > c = 1.76$ ($\alpha = 5\%$). Hypothesis rejected.
9. Hypothesis $\tilde{\mu} = 0$. Alternative $\tilde{\mu} > 0$, $\bar{x} = 1.58$, $t = \sqrt{10} \cdot 1.58/1.23 = 4.06 > c = 1.83$ ($\alpha = 5\%$). Hypothesis rejected.
11. Consider $y_j = x_j - \tilde{\mu}_0$.
13. $n = 8$; 4 transpositions, $P(T \leqq 4) = 0.007$. Assert that fertilizing increases yield.
15. $P(T \leqq 2) = 2.8\%$. Assert that there is an increase.

Problem Set 25.9, page 1111

1. $y = 0.98 + 0.495x$
3. $y = -11{,}457.9 + 43.2x$
5. $y = -10 + 0.55x$
7. $y = 0.5932 + 0.1138x$, $R = 1/0.1138$
9. $y = 0.32923 + 0.00032x$, $y(66) = 0.35035$
13. $c = 3.18$ (Table A9), $k_1 = 43.2$, $q_0 = 54{,}878$, $K = 1.502$, $\text{CONF}_{0.95}\{41.7 \leqq \kappa_1 \leqq 44.7\}$.
15. $y - 1.875 = 0.067(x - 25)$, $3s_x^2 = 500$, $q_0 = 0.023$, $K = 0.021$, $\text{CONF}_{0.95}\{0.046 \leqq \kappa_1 \leqq 0.088\}$

Chapter 25 Review Questions and Problems, page 1111

15. $\hat{\mu} = 20.325$, $\hat{\sigma}^2 = (\frac{7}{8})s^2 = 3.982$
17. $\text{CONF}_{0.99}\{27.94 \leqq \mu \leqq 34.81\}$
19. $c = 14.74 > 14.5$, reject μ_0; $\Phi((14.74 - 14.50)/\sqrt{0.025}) = 0.9353$
21. $2.58 \cdot \sqrt{0.00024}/\sqrt{2} = 0.028$, LCL $= 2.722$, UCL $= 2.778$
23. $\alpha = 1 - (1 - \theta)^6 = 5.85\%$, when $\theta = 0.01$. For $\theta = 15\%$ we obtain $\beta = (1 - \theta)^6 = 37.7\%$. If n increases, so does α, whereas β decreases.
25. $y = 3.4 - 1.85x$